OGAP Fraction Framework – Part to Whole, Equivalence and Magnitude, and Operations (May 11, 2008)

Structures of Fraction Problems

Models
- Area
- Set
- Linear

To solve problems
To understand concepts
To generalize concepts

Wholes
- Same sized wholes
- Different sized wholes
- Given part, find whole

Number of Parts in Whole
relative to the magnitude of the denominator
- Equal
- Multiples
- Factors

In a model or problem situation

Fractions
- Unit fractions
- Non-unit fractions
- Proper fractions
- Improper fractions
- Mixed numbers

Classes of Fractions
- Same numerators, different denominators
- Different numerators, same denominators
- Different numerators and denominators

Partitioning Strategies
- Algorithmic halving (e.g., 1/2, 1/4, 1/8)
- Oddness (e.g., 1/3, 1/5, 1/7)
- Eveness (e.g., 1/6, 1/10, 1/12)
- Composition (e.g., for 12ths partitions into a 3x4 instead of a 1 x 12)

Number Lines
- 0 -1
- Negative to positive
- More than 2 units
- Unpartitioned
- Partitioned

Reasoning Strategies
- Number sense
- Unit fraction
- Extended unit fraction
- Modeling
- Benchmarks/reference points
- Equivalence
- Common denominators
- Density of Fractions

Operations

All Operations
- Estimation
- Number sense
- Modeling
- Equivalence

Multiplication and Division
- Impact of multiplying or dividing by a fraction
- Partitive division
- Quotative division

Evidence of Fractional Reasoning

	Fractional Strategy	Transitional Fractional Strategy	Fractional Strategy with an Error or Miconception	Non-Fractional Reasoning
Across Part to Whole, Equivalence, Magnitude, and Operations	- Uses number sense - Uses efficient strategy	Uses student generated model(s) successfully	**Modeling or partitioning errors:** - Obviously partitions into unequal sized parts - Partitions (draws lines) equal in number to the denominator - Wholes different sizes	Inappropriate whole number reasoninig Inappropriate model, strategy, or operation for problem situation Incorrect notation
Part to Whole	- Uses "out of equal parts" strategy - Identified a part to whole relationship in a model or problem situation	- Uses "out of pieces" strategy - Uses extraneous partitioning - Uses model other than the model given	**(See modeling errors above)**	Partitions only part of a figure Part to part reasoning, not part to whole Does not find fractional part (proper or mixed fractions) Does not find fractional part involving non-adjacent shading
Number Line, Compare, and Order	- Uses number sense/estimation - Uses benchmark reasoning - Uses unit fraction reasoning - Uses extended unit fraction reasoning - Uses equivalence/common denominator - Partitions given model	Uses student generated model(s) successfully	Uses impact of the magnitude of denominator without consideration of the numerator	Does not recognize non-congruent parts as equivalent Identifies fractions as equivalent when they are the same number of parts from whole (e.g., ¾ and 5/6)
	Locates fraction in (about) the correct location relative to the units on the number line: a) using number sense b) partitioning given line c) using measurement	Locates one or more of fractions given on number line, but not all	Fraction located in the correct relative location on a number line, but not accurately On a multiple unit number line - locates proper fraction between 0 -1, but not accurately	Does not place fraction in correct location on number line.
Operations	- Uses efficient algorithm - Uses number sense/estimation	Uses student generated model(s) successfully	Uses appropriate operation or strategy given problem, but solution includes an error (e.g., calculation, equivalence) Fraction not identified in "fair share" problems	Inappropriate model, strategy, or operation for problem situation

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