

## OGAP Multiplication Framework - Problem Structures

Depending upon the strength of multiplicative reasoning students may move back and forth between multiplicative, transitional, additive and non-multiplicative strategies as they interact with different problem structures (e.g., context, magnitude of factors, divisors, or dividends)  
 Kouba, V. & Franklin, K., 1995;  
 VMP OGAP, 2006)

### Problem Situations

- Combinations
- Equal Groups
- Equal Measures
- Equations
- Measure Conversion
- Multiplicative Change
- Multiplicative Comparison
- Part/whole
- Patterns
- Properties
- Rate
- Rectangular Area
- Volume
- (Greer, B. (1992))

## Structures of Problems

<b>Types of Items</b>	Application/Context	<b>Multiplicative Representations</b>	<b>Complexity of Numbers</b>	<b>Number of Factors</b>	<b>Language</b>	<b>Understanding Properties</b>
	Concepts/Property					<b>Factors</b> Single digit Multiple digit Powers of ten Fractions/decimals  <b>Divisors</b> Single digit Multiple Digits Powers of ten Fractions/decimals
<b>Types of Division</b>	Partitive	<b>Multiplicative Representations</b>	<b>Complexity of Numbers</b>	<b>Number of Factors</b>	<b>Language</b>	<b>Understanding Relationships</b>
	Quotative					Equal groups Arrays Area Open area Linear

# OGAP Multiplicative Reasoning Framework - Multiplication

**Multiplicative Strategies**

**Algorithms**

Partial Products  
 $16 \times 4 = 40 + 24 = 64$

Distributive property  
 $16 \times 4 = 4(10 + 6) = 4(10) + 4(6) = 40 + 24 = 64$

Traditional

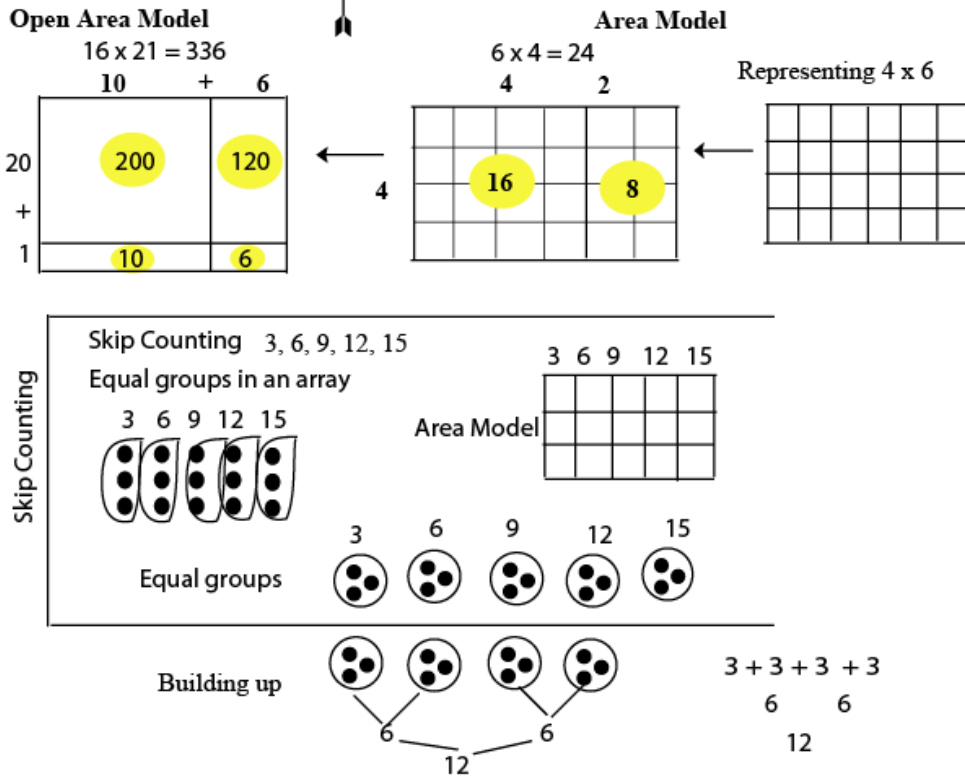
Doubling and Halving  
 $16 \times 4 = 8 \times 8 = 64$

Associative property  
 $8 \times 2 \times 4 = 2 \times 8 \times 4$

Commutative property  
 $16 \times 4 = 4 \times 16$

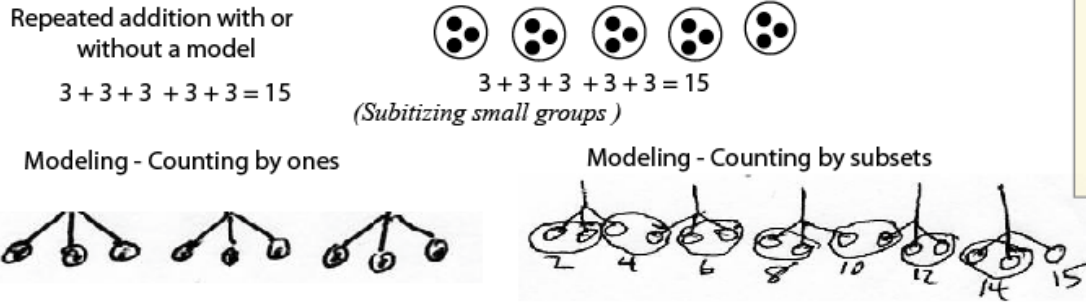
Known or derived fact  $4 \times 6 = 24$

**Transitional Multiplicative Strategies**



*(Unitizing into groups and subgroups)*

**Additive strategies**



Depending upon the strength of multiplicative reasoning students may move back and forth between multiplicative, transitional, and additive strategies as they interact with different problem structures (e.g., context, magnitude of factors, number of factors). (Kouba, V.&Franklin, K., 1995; VMP OGAP, 2006).

**Non-multiplicative Strategies**

- Adds or subtracts factors
- Models factors incorrectly
- Uses incorrect operation
- Not enough information
- Guesses

- Underlying Issues/Errors**
- Misinterprets meaning of quantities
  - Units inconsistent or missing
  - Calculation error
  - Place value error
  - Vocabulary error
  - Property or relationship error
  - Equation error

# OGAP Multiplicative Reasoning Framework - **Division**

## Algorithms

**Multiplicative Strategies**

**Partial quotients**

$$61 \overline{) 686} \begin{array}{l} \underline{610} \quad \times 10 \\ \underline{-76} \quad \times 1 \\ \underline{61} \\ 15 \end{array}$$

**Traditional algorithm**

$$11 \text{ R } 15$$

**Distributive property**  
35 divided by 7 = (21 + 14) divided by 7 = 3 + 2 = 5

**Using the inverse relationship between multiplication and division**

$$24 \text{ divided by } 4 = 4 \times \text{what} = 24$$

**Known or derived fact**

$$24 \text{ divided by } 4 = 6$$



Depending upon the strength of multiplicative reasoning students may move back and forth between multiplicative, transitional, and additive strategies as they interact with different problem structures (e.g., context, magnitude of factors, number of factors). (Kouba, V. & Franklin, K., 1995; VMP OGAP, 2006).

**Transitional Division Strategies**

**Inefficient use of partial quotients**

$$61 \overline{) 256} \begin{array}{l} \underline{61} \quad \times 1 \\ \underline{195} \quad \times 1 \\ \underline{-61} \quad \times 1 \\ \underline{134} \quad \times 1 \\ \underline{-61} \quad \times 1 \\ \underline{-73} \quad \times 1 \\ \underline{61} \quad \times 1 \\ 12 \end{array}$$

**Using multiplication to find a quotient by trial and error**

$$61 \overline{) 256} \begin{array}{l} \underline{244} \\ \underline{12} \\ 4 \text{ R } 12 \end{array} \quad \begin{array}{l} 10 \times 61 = 610 \\ 8 \times 61 = 488 \\ 2 \times 61 = 122 \\ 3 \times 61 = 183 \\ 4 \times 61 = 244 \end{array}$$

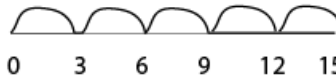
**Modeling in an array to find missing dimension**  $15 \div 5 = 3$



**Skip Counting**

$$15 \div 5 = 3$$

3, 6, 9, 12, 15



**Additive strategies**

**Repeated subtraction or adding up to**

$$12 \div 4 = 3$$

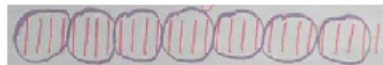
$$\begin{array}{l} 12 - 4 = 8 \\ 8 - 4 = 4 \\ 4 - 4 = 0 \end{array} \quad 4 + 4 + 4 = 12$$

**Sharing out in groups**  $15 \div 5 = 3$



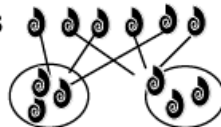
**Modeling**

**Modeling repeated subtraction**



**Sharing out by ones**

$$6 \div 2 = 3$$



## Non-multiplicative Strategies

- Adds or subtracts dividend/divisor
- Models problem incorrectly
- Uses incorrect operation
- Not enough information
- Guesses

## Underlying Issues/Errors

- Misinterprets meaning of quantities
- Misinterprets remainders
- Units inconsistent or missing
- Calculation error
- Place value error
- Vocabulary error
- Property or relationship error

