OGAP Multiplicative Frameworks

Depending upon the strength of multiplicative reasoning students may move up and down between multiplicative, transitional, additive, and non-multiplicative strategies as they interact with different problem situations and problem structures (Kouba & Franklin, 1995; VMP OGAP, 2006).

Problem Contexts

Application/Context
Equal groups
Equal measures
Measure conversions
Multiplicative comparison
Patterns
Rate
Rectangular area
Volume

Concept/Property
Equations
Properties

Problem Structures

COMPLEXITY OF NUMBERS

Types of Items
Application/context
Concept/property

Understanding and Use of Relationships
Addition - Multiplication
Doubling and Halving
Model - Equation
Multiples and Factors
Meanings of remainders
Context - Equation
Context - Model
Powers of ten

Factors
Single digit
Multiple digit
Powers of ten
Fractions/decimals

Language
Natural (e.g., every)
Mathematical (e.g., per)

Types of Division
Partitive
Quotative

Divisors
Single digit
Multiple digit
Powers of ten
Fractions/decimals

Multiplicative Representations
Equal groups
Arrays
Area
Open area
Linear

Understanding and Use of Properties
Associative
Commutative
Distributive
Equality
Identity
Multiplicative Inverse

Number of Factors
Two factors
More than two factors

Problem Situations
Refers to known and unknown information in a problem. For example, in equal group problems there might be an unknown product, or an unknown number of equal groups, or an unknown quantity in a group. See page 4 for some examples. Also see the CCSSM page 89.

About OGAP Frameworks

OGAP Frameworks are based on mathematics education research on how students learn specific mathematics concepts, errors students make, and pre-conceptions or misconceptions that may interfere with learning new concepts or solving related problems.

There are three major elements to an OGAP Framework that should be considered when analyzing student work or making instructional decisions:

1. Problem contexts
2. Problem structures
3. Evidence in student work

This page identifies problem situations and problem structures for multiplication and division problems. Pages 2 and 3 are tools to help teachers classify evidence in student work, including classroom discussions, and make instructional decisions. Page 4 has samples of different problem contexts and situations.

For students to become strong multiplicative reasoners they must interact with a range of problem situations and problem structures. The CCSSM specifically identifies problem contexts at targeted grades on a progression from equal groups and measures, and area situations at grades 2 and 3 to measurement conversions, area, volume, multiplicative patterns, and multiplicative comparison problem situations at grades 4 and 5. This progression,among other things, is designed to prepare students to engage in proportional situations using multiplicative reasoning.

Consistent with the CCSSM the OGAP Frameworks on pages 2 and 3 show a progression from the link between repeated addition and multiplication in an equal groups model to the development of efficient and generalizable multiplicative strategies through the open area model, and understanding of place value, properties of operations, and relationships.

As students interact with new concepts, new problem situations, new structures, and more complex problem solving situations they may move back and forth between multiplicative, transitional, additive, and non-multiplicative strategies. This is important evidence to use for instructional decision-making. For example, a student may consistently solve equal group problems using a Multiplicative Strategy regardless of the complexity of the numbers, but you may find that the same student adds factors (Non-Multiplicative Strategy) when solving multiplicative change problems.

This is a derivative product of the Vermont Mathematics Partnership Ongoing Assessment Project (OGAP) which was funded by NSF (EHR-0227057) and the US DOE (S366A020002).

OGAP Multiplicative Reasoning Framework – Multiplication

Multiplicative

Transitional

Additive

Multiplicative Strategies

Algorithms

Partial Products

$$\begin{array}{r} 16 \\ \times\ 42 \\ \hline 12 \\ 20 \\ 240 \\ 400 \\ \hline 672 \end{array}$$

Traditional

$$\begin{array}{r} {\scriptstyle 21} \\ 16 \\ \times\ 42 \\ \hline 32 \\ 640 \\ \hline 672 \end{array}$$

Distributive Property

$4 \times 16 = 4(10 + 6)$
$= 4(10) + 4(6)$
$= 40 + 24$
$= 64$

Associative Property

$(8 \times 2) \times 5 = 8(2 \times 5)$
$= 8 \times 10$
$= 80$

Doubling & Halving

$16 \times 4 = 8 \times 8$
$= 64$

Known or Derived Fact

$4 \times 6 = 24$

Commutative Property

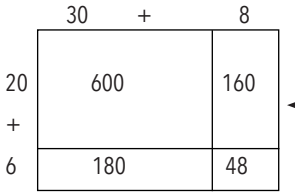
$16 \times 4 = 4 \times 16$

Powers of Ten

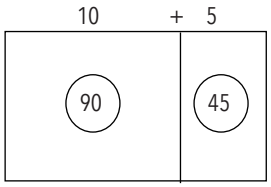
$5 \times 400 = 5 \times 4 \times 10 \times 10$

Transitional Strategies

Open Area Model *Considers both dimensions of an array or area model*

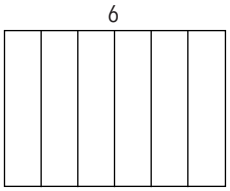


38 x 26 = 988



15 x 9 = 135

Area Model



6 x 4 = 24

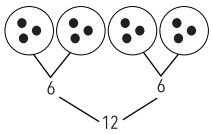
Considers BOTH dimensions of an array or area model, moving away from needing to see every square unit

Early Transitional Strategies

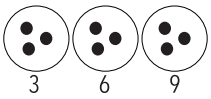
Skip Counting 3, 6, 9, 12, 15

Building up

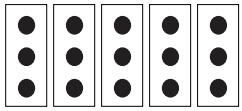
$3+3+3+3$
$6+6$
12



Skip Counting with a Model

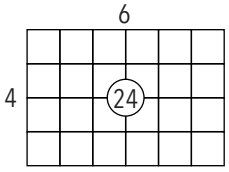


Equal groups in an array



Considers only ONE dimension of an array or area model

Area Model – 6 x 4 = 24

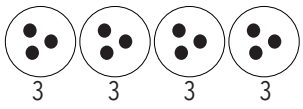


Considers BOTH dimensions of an array or area model

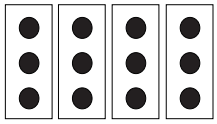
Additive Strategies

Repeated addition with or without a model – 3 x 4=12

$3+ 3 + 3 + 3 = 12$



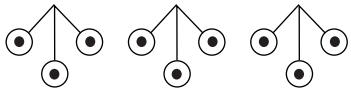
Subitizing in small groups



3 + 3 + 3 + 3

Early Additive Strategies

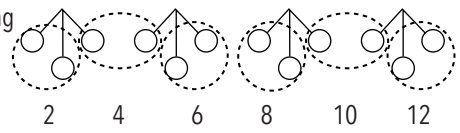
Modeling, counting by ones



Inconsistent Grouping

Modeling, counting by subgroups

3 x 4=12



Non-Multiplicative Strategies

Adds or subtracts factors
Models factors incorrectly
Guesses
Uses incorrect operation
Not enough information
Uses procedures incorrectly

Underlying Issues/Errors

- Doesn't consider reasonableness of solution
- Error in: calculation, place value, vocabulary, property or relationship, equation, or model
- Misinterprets the remainders
- Units inconsistent or missing

Applies understanding of place value, properties, and relationships

Unitizes into groups and sub-groups

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OGAP Multiplicative Reasoning Framework – Division

Multiplicative

Transitional

Additive

Multiplicative Strategies

Partial Quotients

17 | 585
340 x 20
245
170 x 10
75
68 x 4
7 34 r 7

Algorithms

4
10
20 34 r 7
17 | 585
340
245
170
75
68
7

Traditional

34 r 7
17 | 585
51↓
75
68
7

Distributive Property

$35 \div 7 = (21 + 14) \div 7 = 3 + 2 = 5$

Treats the remainder appropriately given problem situation

Inverse relationship between multiplication and division

$35 \div 7 = 5 \quad 7 \times ? = 35$

Known or Derived Fact

$21 \div 7 = 3$

Transitional Strategies

Inefficient partial quotients

61 | 756 — 12 R 24
183 x 3
573
183 x 3
390
183 x 3
207
183 x 3
24

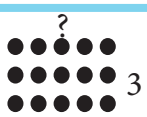
Trial and error to find a quotient

61 | 756
732
24
12 R 24

2 x 61 = 122
20 x 61 = 1220
4 x 61 = 244
8 x 61 = 488
15 x 61 = 915
10 x 61 = 610
12 x 61 = 732

Early Transitional Strategies

Models in an array to find missing dimensions



$15 \div 3 = ? \quad 3 \times ? = 15$

Skip Counts to find the number of "skips" with and without a model

$15 \div 3 = 5$ 3, 6, 9, 12, 15 (5 skips)

Additive Strategies

Repeated subtraction or adding up to

61 | 350 — 5 R 45
61
289
61
228
61
167
61
106
61
45

350
-305
r 45

61
+61
122
+61
183
+61
244
+61
305
+61
366

Sharing out in equal groups as repeated subtraction or addition

There are 8 cookies to share equally with 4 children. How many cookies does each child get?

8 cookies ÷ 4 children = 2 cookies per child

Represents the 4 children with circles and then fills them equally

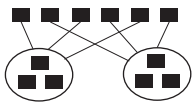
Twenty-four cookies were put into bags of 4 cookies each. How many bags were filled?

24 cookies ÷ 4 cookies per bag = 6 bags

Pulls out 4 cookies at a time until 24 cookies are used

Early Additive Strategies

Sharing out by ones



Sharing out randomly by subsets

Sharing 4, then 2, then 5, and so on

Non-Multiplicative Strategies

Adds or subtracts dividends/divisors
Models problem incorrectly
Guesses
Uses procedures incorrectly
Uses incorrect operation
Not enough information

Underlying Issues/Errors

- Doesn't consider reasonableness of solution
- Error in: calculation, place value, vocabulary, property or relationship, equation, or model
- Misinterprets the remainders
- Units inconsistent or missing

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OGAP Sample Problem Contexts and Situations

Important: The sample problem contexts and situations do not include the full range of each problem situation or context.

Multiplication Examples	Division Examples How many in each group? (partitive) How many groups? (quotative)
Equal group, measurement conversion, equal measure, and rate problems involve applying a rate. number of groups/measurements/quantities x rate = total number Multiplicative comparison and multiplicative patterns involve a multiplicative scale factor. original x scale factor/multiplier = result Area and volume problems involve using dimensions in either an area or volume situation. *Problem situations are identified with an * and are in italics in the examples below.*	
Equal Group **Unknown product (total number of crayons)* Mark bought 12 boxes of crayons. Each box contained 8 crayons. How many crayons were there all together? 12 boxes x 8 crayons per box = ? crayons	** Unknown number in each group (crayons per box)* Mark had a box of 64 crayons. He shared the crayons equally with 4 people. How many crayons did each person get? (partitive) 64 crayons = 4 people x ? crayons per box
Equal Measures ** Unknown product(total length in inches)* It takes 14 inches of ribbon to make one bow. How many inches of ribbon will it take to make 7 bows? 7 bows x 14 inches per bow = ? inches	** Unknown number of groups (number of designs)* Sam ha s 15 yards of material. He is making a design that needs 3 yards per design. How many designs can Sam make? (quotative) 15 yards ÷ 3 yards per design = ? designs
Measurement Conversion **Unknown product (length in inches)* Tammy is 5 feet tall. How many inches tall is Tammy? 5 feet x 12 inches/foot = ? inches	
Rates **Unknown product (total dollars)* Sam works at the grocery store. He is paid $7.00 per hour. He worked 22 hours last week. How much money did Sam earn last week? 22 hours x $7.00/hour = ? dollars	** Unknown rate (dollars per hour)* Sam earned $154.00 last week. He worked 22 hours. How much did Sam earn per hour? (partitive) $154.00 = 22 hours x ? dollars per hour
Multiplicative Comparison ** Larger unknown (height in inches)* The students in Mrs. Gilbert's class planted bean and corn seeds. The bean plants grow 3 times faster than the corn plants. When the corn plants measure 2 inches, how tall will the bean plants be? 2 inches x 3 = ? height of bean plants	** Multiplier unknown (scale factor unknown)* Bill's garden is 240 square feet. Leslie's garden is 20 square feet. How many times greater is Bill's garden than Leslie's garden? (quotative) 240 square feet ÷ 20 square feet = ? times bigger
Patterns ** Larger unknown (length)* A 5-inch piece of elastic is stretched 3 times its length. How long is the elastic after it is stretched? (4 times, 5 times, *n* times) 5 inches x 3 = ? (total length)	
Area ** Unknown product (area in square feet)* Linda's kitchen floor measures 12 feet by 7 feet. How many tiles (1 square foot) are needed to cover the floor? 12 feet x 7 feet = ? (total area in square feet)	** Unknown factor (length of kitchen floor)* Linda's kitchen floor is 150 square feet. The length of one dimension is 10 feet. What is the length of the other dimension of the kitchen floor? 150 square feet ÷ 10 feet = ? (length of other dimension in feet)
Volume ** Unknown volume (volume of water the tank can hold)* The dimensions of a fish tank are 10 inches x 12 inches by 18 inches. If you filled the tank to the top, how much water would be in the tank? 10 inches x 12 inches x 18 inches = ? amount of water in tank	** Unknown factor (height of container)* A rectangular container holds 250 cubic inches of liquid. the base of the container is 10 inches and the depth 5 inches. What is the height of the container? 10 inches x 5 inches x ? inches = 250 cubic inches
OGAP Equation Example: 6 x 5 = 30; Write a story problem that goes with this equation. OR match a story problem to an equation.	**OGAP Property Example**: Ann knows the answer to 9 x 5. Explain how can she use this information to solve 45 ÷ 9?