Facilitating Use of Formative Assessments: Multiplicative Reasoning—Ongoing Assessment Project
Bob Laird, Vermont Mathematics Institute, University of Vermont (rlaird@uvm.edu)
Marge Petit, Marge Petit Consulting, MPC (mpetit@gmavt.net)

OGAP Sites:
Vermont
Alabama
Michigan
Ohio
Amman, Jordan
Nebraska

2012 NCSM Annual Meeting
In the end – it is the evidence of student thinking not just from assessment questions, but also from classroom discussions and activities that informs instructional decision making.
Take Aways!

- **Teacher knowledge** about the research/learning trajectories is fundamental – this involves a real commitment to PD, NOT just creating tools and materials, but substantive professional development.

- **Evidence of Student Thinking** - it is the evidence of student thinking not just from assessment questions, but from classroom discussions and activities that informs instructional decision making.

- **Formative assessment** is a powerful tool when it is implemented systematically and intentionally coupled with the above.

- The **CCSS** and OGAP Framework
In 1 hour...

What can be done ....

• …provide participants with the big idea of OGAP and introduce to the OGAP Multiplicative Framework

What **cannot** be done …

• … provide participants with a deep understanding of the details and potential implications of OGAP and the research related to students developing their multiplicative reasoning

• … be sure that participants understand the difference between formative and summative assessment.
OGAP is a systematic and intentional formative assessment system in mathematics.

- Gathering information about pre-existing knowledge through the use of a pre-assessment;
- Analysing pre-assessment to guide unit planning; and
- A continuous and intentional system of instructing, probing with instructionally embedded questions, analysis, and instructional modification.

Grades 2 - 8
- Fractions
- Multiplicative reasoning
- Proportionality
In place and in use for all 3 mathematical topics

- Pre-assessments and ongoing questions
- Tools and strategies to analyze student work
- Professional development workshop materials and resources to communicate research and support the use of OGAP formative assessment system
OGAP was Developed Based on Four Principles
Principle # 1: Build on pre-existing knowledge  (How People Learn (2000) National Research Council)

Principle # 2: Learn (and assess) for Understanding  

Principle # 3: Use Frequent Formative Assessment  
(Inside the Black Box, (2001) Black, P, and Wiliam, D.)

Principle # 4: Build Assessment on Mathematics Education Research  (Knowing What Students Know (2001) National Research Council)
It is not formative assessment alone OR knowledge of cognitive research alone…

…but the marriage of the two that empowers teachers
In design of materials
- formative assessment items
- professional development materials (case studies, activities, essays)
- Book and articles

In work with educators
- analyze student work
- inform instructional decisions
- help understand the purposes of activities in mathematics programs

Hundreds of research articles distilled into frameworks and used
Research to Practice
Teachers say understanding the math education research help them...

- Understand the purposes of activities in math programs;
- Understand evidence in student work used to inform instruction;
- Strengthen and focus first wave instruction;
- Respond to evidence in student work as instruction proceeds.
Solve the following problems 3 different ways

A) How many wheels do 5 tricycles have?

B) How many wheels do 29 tricycles have?
Review the Framework
Intro to OGAP Framework: About the problem

- What is the problem situation?
- What are other problem structures to consider?
- What are strategies that were used? Where are they on the OGAP Framework?

Depending upon the strength of multiplicative reasoning, students may move back and forth between using multiplicative, transitional, additive, and non-multiplicative strategies as they interact with different problem structures and problem situations. (Kouba & Franklin, 1995; VMP OGAP, 2006)
How many wheels do 29 tricycles have?
One tricycle has three wheels.

Write an equation to match this picture.

A class has set a goal that each student will read 45 pages this week. There are 16 students in the class. How many pages will they have read altogether by the end of the week?
Analyzing student work – the OGAP Sort
## Problem Situations

<table>
<thead>
<tr>
<th>Grade 2</th>
<th>Equal groups</th>
<th>Repeated addition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 3 (factors within 100)</td>
<td>Equal groups, arrays, equal measures, beginning area</td>
<td>Properties of operations, drawings, equations</td>
</tr>
<tr>
<td>Grade 4 (1 digit x 4 digit, and 2 digit x 2 digit)</td>
<td>Multiplicative comparison, measurement conversion within systems, area</td>
<td>Place value and properties... using equations, rectangular arrays, and/or area model, equations</td>
</tr>
<tr>
<td>Grade 5 (fluently)</td>
<td>Scaling (multiplicative change), area, volume, patterns, conversions between systems</td>
<td>Standard algorithm, equations</td>
</tr>
</tbody>
</table>

### Link to the OGAP Framework

- Whole Number Multiplication
- Link to the OGAP Framework
OGAP is a systematic and intentional formative assessment system in mathematics.

- Gathering information about pre-existing knowledge through the use of a **pre-assessment**;

- **Analyzing pre-assessment** to guide unit planning; and

- A **continuous and intentional system** of instructing, probing with instructionally embedded questions, analysis, and instructional modification.

Grades 2 - 8
- Fractions
- Multiplicative reasoning
- Proportionality
Questions and Answers
What do *teacher leaders and teachers* say about their experience in relationship to the stated goals and the use of OGAP formative assessment system?

Results based on a spring 2007 online survey
Expertise for analyzing student work (for evidence of developing understanding, common errors and misconceptions)…

Before and After Experience

Expertise for Analyzing Student Work (n=104)

Number of Teachers

1 (low) 2 3 (moderate) 4 5 (high)

Before
After

0 10 20 30 40 50 60
Expertise in using evidence in student work to inform instruction…
Understanding purposes of activities in mathematics program…

Before and After Experience

Expertise understanding purposes of math program
(n=104)

Number of Teachers

Before
After

1 (low)
2
3 (moderate)
4
5 (high)
Fraction content knowledge…

Before and After Experience

Fraction content knowledge (n=104)

Number of Teachers

Before

After

1 (low)  2  3 (moderate)  4  5 (high)
Pre-post Question – Pilot OGAP Teacher Assessment (2007)

Provide three strategies students can use to solve this problem. Provide examples.

1) Which fraction is closest to 1? Show your work.

\[
\frac{1}{2} \quad \frac{7}{9} \quad \frac{11}{13} \quad \frac{1}{6}
\]
Sample Teacher Responses

Pre-assessment Q1 A

1. \( \frac{1}{2} = \frac{117}{234} \), \( \frac{7}{9} = \frac{182}{234} \), \( \frac{11}{13} = \frac{198}{234} \)
   \( \frac{1}{6} = \frac{39}{234} \). \( \therefore \frac{11}{13} \) is closest to 1

2. Use fraction bars kit provided, (ninthths + thirteenth are in it)

3.

Post-assessment Q1 A

1. Unit fractions: \( \frac{1}{2}, \frac{1}{6} \)
   Sixth are smaller parts than halves.

2. Use of area models

3. Use \( \frac{1}{2} \) benchmark.
   Using unit fraction reasoning, \( \frac{1}{6} \) is smaller than \( \frac{1}{2} \).
   \( \frac{7}{9} \) and \( \frac{11}{13} \) are greater than \( \frac{1}{2} \).
   (Continue on back as needed)

\( \frac{11}{13} \) is \( \frac{2}{13} \) away from 1 whole.
\( \frac{7}{9} \) is \( \frac{2}{9} \) away from the 1 whole.
Since 13ths are smaller, \( \frac{11}{13} \) is closer to 1.
Findings (Petit-Cunningham, 2008)

- Teacher leaders increased the range of strategies that they used pre to post to solve the two problems.
- Mentees also increased the range, but to a lesser degree

<table>
<thead>
<tr>
<th>Mentors and Mentees Pre - Post Teacher Assessment</th>
<th>Pre mean</th>
<th>Post mean</th>
<th>T-test (p-) Significance (p&lt; 0.05)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mentors (n=25)</td>
<td>6.16</td>
<td>9.8</td>
<td>3.52E-08</td>
</tr>
<tr>
<td>Mentees (n= 42)</td>
<td>5.6</td>
<td>7.9</td>
<td>7.73E-06</td>
</tr>
</tbody>
</table>
Related Publications

- Petit, Laird, & Marsden (September, 2010). They get fractions as pies – but now what?. Mathematics in the Middle School, NCTM, Reston, Virginia.
For more information go to margepetit.com or contact...

**Bob Laird**, Vermont Mathematics Institute, University of Vermont  
(rlaird@uvm.edu)

**Marge Petit**, Marge Petit Consulting, MPC (mpetit@gmavt.net)

[www.margepetit.com](http://www.margepetit.com)

**Beth Hulbert**, Barre City and Town Schools, Barre VT  
(hulbertbj1@myfairpoint.com)
For OGAP References go to:

- [http://margepetit.com/](http://margepetit.com/)
Bringing OGAP to your school, district, or state involves…

**Significant Professional Development by OGAP team and ongoing support system at the school level**

- In an understanding of formative assessment
- In the use of OGAP formative assessment materials and processes.
- On the substance of the math education research that is foundational to the OGAP materials and processes.
- Use of the materials “real time” with students with links to mathematics programs.

**Tools and Resources to support system**

- Some pre-assessments and ongoing items
- Strategies and related tools for analyzing student work and making instructional decisions
OGAP Development Team and National Advisory Board

Vermont OGAP Design Team

- Leslie Ercole, VMP
- Linda Gilbert, Dotham Brook School
- Kendra Gorton, Milton Elementary School
- Steph Hockenbury, Chamberlin School
- Beth Hulbert, Barre City Elementary and Middle School
- Amy Johnson, Milton Elementary School
- Bob Laird, VMP
- Ted Marsden, Norwich University
- Karen Moylan, Former VMP
- Cathy Newton, Dotham Brook School
- Susan Ojala, Vermont Mathematics Initiative
- Nancy Pollack, Chittenden East
- Marge Petit, Marge Petit Consulting, MPC
- Regina Quinn, VMP
- Loree Silvis, VMP
- Krisan Stone, VMP
- Corrie Sweet, Former VMP
- Tracy Thompson, Ottauquechee School
- Jean Ward, Bennington Rutland Supervisory Union
- Rebecca Young, Hardwick Schools

Plus about 250 Vermont and Alabama teachers and teachers and about 5000 students who participated in OGAP Exploratory Studies and 2006-2008 scale-up

OGAP National Advisory Board

- **Mary Lindquist**, Callaway Professor of Mathematics Education, Emeritus; Past President of the National Council of Teachers of Mathematics
- **Ed Silver**, University of Michigan
- **Judith Zawojewski**, Illinois Institute of Technology

OGAP Sites:
- Vermont
- Alabama
- Michigan
- Ohio
- Amman, Jordan
- Nebraska